# Understanding Bohmian Mechanics – A Dialogue

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## First Day: Fundamentals

Alice: What does Bohmian mechanics exactly say?

Bob: It describes the motion of $N$ point particles in usual three-space. Every particle $i$ has at every time $t$ some definite position $Q_i \in \mathbb{R}^3$. The motion satisfies this first-order differential equation:

$$\dot{Q}_i = \frac{\hbar}{m_i} \Im \frac{\nabla_i \Psi(Q_1 \dots Q_N, t)}{\Psi(Q_1 \dots Q_N, t)} \tag{1}$$

where $\Im$ means 'imaginary part' and $\Psi$ is some time-dependent complex-valued function on configuration space $\cong \mathbb{R}^{3N}$ that satisfies Schrödinger's equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \nabla_i \cdot \nabla_i \Psi + V(q_1 \dots q_N)\Psi \tag{2}$$

where $V$ is the potential energy. (We denote the variables $\Psi$ depends on by $q$ and the actual positions of the particles by $Q$).

Alice: And this is intended to replace non-relativistic quantum mechanics?

Bob: Yes. The idea is that Bohmian mechanics is the true quantum mechanics. The $\Psi$ function is just the very same wave function you know from quantum mechanics, and the positions of the particles are just the same you would find if you performed a position measurement in quantum mechanics.

Alice: So the Bohmian answer to 'wave or particle?' is 'wave and particle!'

Bob: Yes.

Alice: But it's very different from usual quantum mechanics on the conceptual level, isn't it? Indeed, it's not a quantum theory at all, it's a classical theory.

Bob: It is indeed very different from usual quantum mechanics on the conceptual level. Usually, it is assumed that quantum particles don't have trajectories. But, as we will see soon, Bohmian mechanics is in perfect agreement with all probabilistic predictions of quantum mechanics. So you are mistaken when thinking Bohmian mechanics is not a quantum theory; you must remember that its empirical implications agree with quantum mechanics (whenever quantum mechanics is unambiguous), and disagree with Newtonian mechanics. A corollary of this agreement is that Bohmian mechanics is confirmed by experience. In particular, the mere existence of Bohmian mechanics proves that the alleged non-existence of trajectories cannot be concluded from the experimental facts.

Alice: You will have to explain the agreement with the predictions of quantum mechanics. But before this, I have some questions on the dynamics. Apparently, you have to assume that the wave function is not merely square-integrable, but indeed differentiable.

Bob: I do assume that the wave function is differentiable (except perhaps a few exceptional configurations).

Alice: For all times?

Bob: For all times. For a reasonably large class of potentials (including Coulomb), there is a dense subspace in $L^2$ of wave functions that will be differentiable for all times (with few exceptional configurations).

Alice: And the equation of motion is ill-defined for all nulls of the wave function. What if your trajectory $(Q_1(t) \dots Q_N(t))$ runs into a null?

Bob: It has been proved [2] that for almost all initial configurations (according to an appropriate measure), and, of course, for a reasonable class of wave functions, the equation of motion has unique global (i.e. for all $t$) solutions. Hence, Bohmian trajectories almost-surely don't run into the singularities of the velocity field, i.e., the nulls or non-differentiable points of the wave function.

Alice: What is this appropriate measure?

Bob: The natural measure for counting initial configurations (i.e., for talking about the size of a set of initial configurations) for the equation of motion with wave function $\Psi(t=0)$ is

$$|\Psi(q_1 \dots q_N, 0)|^2 d^{3N}q \tag{3}$$

where $d^{3N}q$ is the usual Lebesgue volume measure on configuration space. This induces a measure on the set of solution curves $(Q_1(t) \dots Q_N(t))$ of the equation of motion.

Alice: Why don't we simply count initial conditions by Lebesgue measure?

Bob: We might in fact choose any density function $\rho(q_1, \dots, q_N)$ at time 0. The dynamics will transport this to a density function $\rho(q_1, \dots, q_N, t)$ at time $t$ via the continuity equation

$$\frac{\partial}{\partial t}\rho + \sum_i \nabla_i \cdot \left(\rho \frac{\hbar}{m_i} \Im \frac{\nabla_i \Psi}{\Psi}\right) = 0.$$

But this is arbitrary whereas (3) is a naturally selected measure: Note that if you start with Lebesgue measure, i.e. $\rho = 1$, at some time, the measure will cease to be Lebesgue at other times. So when starting with Lebesgue, you arbitrarily prefer some point in time. Not so with the measure (3). The measure $|\Psi(0)|^2 dq$ is transported by the dynamics to the measure $|\Psi(t)|^2 dq$. This can be easily checked using the Schrödinger equation by proving the continuity equation

$$\frac{\partial}{\partial t}|\Psi|^2 + \sum_i \nabla_i \cdot \left(|\Psi|^2 \frac{\hbar}{m_i} \Im \frac{\nabla_i \Psi}{\Psi}\right) = 0.$$

In other words, the Bohmian velocity $(\hbar/m_i)\Im(\nabla_i\Psi/\Psi)$ equals $j/|\Psi|^2$ where $j$ is what is commonly known as the probability current density of the wave function.

Alice: So what you're saying is, the only obvious way to define a measure on the set of solution curves $(Q_1(t) \dots Q_N(t))$ without preferring some point in time is by (3).

Bob: Precisely.

Alice: Quantum mechanics says $|\Psi(q,t)|^2$ gives the probability density of finding the particle at position $q$ when measuring the position at time $t$. If position measurements simply reveal the Bohmian positions, the Bohmian positions must be random and distributed according to $|\Psi|^2 d^{3N}q$.

Bob: Remember that the wave function we were talking of is the wave function of all particles in the universe. What we have to consider is an ensemble of small subsystems, all within the same universe, and all having the same subsystem wave function $\psi$. It has been shown [6] that for the overwhelming majority (according to the measure (3)) of possible initial configurations of the Bohmian universe the positions of these subsystems look as if random and $|\psi|^2$-distributed. Obviously, there's not much sense in saying the initial configuration of our single universe was random (that's as if we assumed that Zeus, before creating the world, threw a die for determining the dimension of space). To sum up: for a subsystem with wave function $\psi$ we may always assume the configuration to be random and $|\psi|^2$-distributed. This statement is called the Quantum Equilibrium Principle.

Alice: What about the collapse or reduction of the wave packet? Eq. (2) says there's no collapse.

Bob: True. It says there's no collapse for the wave function of the universe. Indeed it is hard to believe that the Schrödinger equation sometimes ceases to hold. Let me derive the collapse of the subsystem wave function from Bohmian mechanics.

Alice: You mean, you can *derive* the collapse from eq.s (1) and (2)? It is well known that the collapse is non-unitary and therefore in conflict with the Schrödinger evolution!

Bob: We *can* derive the collapse. You will see. For simplicity, we consider a 'measurement' with only two possible outcomes. And let's, for simplicity, first suppose a special form of the wave function of the universe, $\Psi = \psi \otimes \phi \otimes \Phi$ where $\psi$ is the wave function of the subsystem to be 'measured', $\phi$ is the wave function of the measuring apparatus, and $\Phi$ is that of the rest of the world. $\Phi$ will be irrelevant to our discussion, so we ignore it here.

Alice: Because, as long as $\Psi$ is a product $\Psi = (\text{something}) \otimes \Phi$, eq. (1) implies the motion of the subsystem and apparatus particles is independent from what's happening outside.

Bob: Yes. Suppose $\hat{U}$ is the unitary that represents the time evolution of the wave function during the 'measurement' process.

Alice: Wait a second: why do you always put these quotation marks around the word 'measure'?

Bob: Because it is very unclear whether anything is actually being measured during what is usually called a 'measurement'. I'll return to this point later.

Alice: Hm. Go on.

Bob: Suppose $\phi_0$ is the wave function of the apparatus before the measurement, say $\phi_1$ is that corresponding to the result 1, and $\phi_2$ is that corresponding to result 2. If $\psi_1$ is the eigenfunction to result 1 and $\psi_2$ the eigenfunction to result 2, we expect that

$$\begin{aligned} \hat{U}\psi_1 \otimes \phi_0 &= \psi_1 \otimes \phi_1, \\ \hat{U}\psi_2 \otimes \phi_0 &= \psi_2 \otimes \phi_2. \end{aligned}$$

Now if $\psi = c_1\psi_1 + c_2\psi_2$ is not an eigenfunction of the self-adjoint operator (the 'observable') corresponding to this 'measurement', then linearity of the Schrödinger equation implies

$$\hat{U}\psi \otimes \phi_0 = c_1\psi_1 \otimes \phi_1 + c_2\psi_2 \otimes \phi_2. \tag{4}$$

$\phi_1$ and $\phi_2$ will have very disjoint configurational support, i.e., $\phi_1$ and $\phi_2$ are supported by some sets $S_1$ and $S_2$ respectively in the configuration space of the apparatus particles (well, $\phi_1$ will not really be zero outside $S_1$ but very close to zero, such that, say, 99.9% of $|\phi_1|^2$ will be concentrated in $S_1$; similar for $\phi_2$), and these two sets will not only be disjoint, but very far apart in configuration space, as they are macroscopically distinct.

Alice: Say, for example, if the result is displayed by the position of a pointer ($10^{23}$ particles) on a scale, all configurations in $S_1$ will have positions of all pointer particles close to 1, so elements of $S_1$ and $S_2$ differ by one length unit in at least $10^{23}$ variables.

Bob: Yes. For all practical purposes, it will be impossible to effectuate any interference between the two wave packets in the right hand side of (4), because for interference, the supports of the two packets have to overlap in configuration space.

Alice: I see.

Bob: So far we discussed only the wave function. Now in Bohmian mechanics, the configuration point of subsystem + apparatus will be, thanks to the Quantum Equilibrium Principle, random and distributed according to $|c_1\psi_1 \otimes \phi_1 + c_2\psi_2 \otimes \phi_2|^2$, which for disjointness of supports $\approx |c_1|^2|\psi_1|^2|\phi_1|^2 + |c_2|^2|\psi_2|^2|\phi_2|^2$. Therefore, the configuration point will reside

in the set {subsystem configurations} $\times S_1$ with probability $|c_1|^2$, and
in the set {subsystem configurations} $\times S_2$ with probability $|c_2|^2$

(note that this coincides with the probability predictions of quantum mechanics); furthermore, if the configuration point resides in the first set, the output of the apparatus will (unambiguously) read '1'.

Alice: And in this case, where is the collapsed wave function of the subsystem after the measurement?

Bob: The future motion of the configuration point will only depend on the first wave packet $c_1\psi_1 \otimes \phi_1$ since, as you can see in eq. (1), the velocity only depends on the value of the wave function and its derivatives *at the configuration point* and the two wave packets never meet again.

Alice: Aha. I can read off from eq. (1) that the factor $c_1$ is irrelevant for the motion, so the subsystem behaves as if it had wave function $\psi_1$.

Bob: Yes.

Alice: But somehow, I missed the point where the collapse comes about.

Bob: If $x, y, z$ are the configuration of the subsystem, the apparatus and the rest of the world, respectively, and $x(t), y(t), z(t)$ is the solution of eq. (1), we might call $\Psi(-, y(t), z(t), t)$ the *conditional wave function* of the subsystem. As long as there is no interaction between the subsystem and anything else, the conditional wave function obeys itself a Schrödinger equation, but ceases to do so during interaction. The conditional wave function collapses, but not so the wave function of the universe.

Alice: What happens to the second wave packet?

Bob: It leads an empty life. It evolves according to Schrödinger's equation, but it doesn't influence the configuration.

Alice: But if $\Psi$ never collapses, it isn't a product $\psi_1\otimes$(something) after the experiment. And we assumed it is a product in the beginning of our discussion of the measurement process. So you can't iterate your argument.

Bob: It isn't necessary to assume $\Psi$ is a product. We might have allowed a number of empty wave packets somewhere far away in configuration space. If $\Psi = \psi \otimes \phi \otimes \Phi + \Psi^{\perp}$, and if the support of $\Psi^{\perp}$ is macroscopically disjoint from that of $\psi \otimes \phi \otimes \Phi$ (which contains the configuration point), then our discussion still applies. In this case $\psi$ is called the *effective wave function* of the subsystem, and $\psi_1$ is the effective wave function of the subsystem after the 'measurement'.

Alice: If I understood you properly, the outcome of the measurement in general depends on the microstate, i.e., the configuration + wave function, of the measurement apparatus. In particular, it depends on the details of $\phi$ which are subject to thermal fluctuations.

Bob: In principle, yes. But for practically relevant experiments, it turns out that the configuration of the apparatus and the details of its wave function don't influence the outcome. The origin of the randomness is the unknown subsystem configuration. But indeed, different experimental arrangements corresponding to the same self-adjoint operator may lead to different outcomes for the same $\psi$ and subsystem configuration.

Alice: So the outcome can't be predicted given a self-adjoint operator and the state (configuration + wave function) of the subsystem?

Bob: In many cases, it can't. That's why 'measurement' is quite a misnomer in this context because it isn't at all a property of the subsystem that is being 'measured'.

Alice: According to Bohmian mechanics. But in other interpretations ...

Bob: At least you don't know in general. Ask yourself how you know that a different apparatus ('measuring' the same 'observable') acting on the same subsystem wouldn't have given a different 'measurement' result.

Alice: I'll have to think about this. Quantum mechanics never understood 'measurement' in the sense of simply revealing a preexisting quantity, but rather of forcing Nature to choose a value.

Bob: All the more 'measurement' is a misnomer. It's a rhetorical trick to pretend a meaning in the outcomes which in general they don't have. Nobody would call dicing a measurement.

Alice: What about the famous quantum paradoxes in Bohmian mechanics?

Bob: No paradoxes (cf. [1]). Bohmian mechanics is given by the eq.s (1) and (2), so there can't be any paradoxes.

## Second Day: Bohmian vs. Orthodox

Alice: I see that Bohmian mechanics is a *possible* explanation of the quantum world. But the particle trajectories can't be observed!

Bob: The word 'observe' is somewhat ambiguous. Strictly speaking, in a Bohmian universe, the particle paths actually *can* be observed. Let's consider, e.g., a single particle, say in a double slit experiment. We finally observe the position of arrival of every single particle on the screen and, since the equation of motion is first-order, can compute the entire trajectory from this position. For instance, we can decide whether the particle passed the left or the right slit, without disturbing the interference pattern: for symmetry reasons, all particles which passed the left slit hit the left half of the screen while those which passed the right slit hit the right half of the screen.

Alice: But your last proposition cannot be tested empirically.

Bob: It cannot be tested empirically. But it's common to physical theories to have implications that cannot be tested empirically.

Alice: I didn't have in mind that you 'observe' the trajectory by calculating it.

Bob: Most observations, be it the mass of the sun or the charge of the electron, are not done directly, but involve calculations. I understand, of course, that you had in mind detecting the particle's position every tenth of a second. But the interaction involved with this detection will influence the particle's future motion, so we won't see the trajectory the particle would have followed if its position hadn't been detected. It's well known that detecting the particle at the slits of a double-slit experiment will make the interference fringes disappear.

Alice: Hence the trajectory cannot be seriously observed, and the equation of motion cannot be tested directly.

Bob: Neither can the Schrödinger equation as we can't observe wave functions.

Alice: Can't we observe wave functions?

Bob: We can't. Assume I prepared an atom with a certain wave function I chose, and I give it to you. You can't find out the wave function if I don't tell you.

Alice: I see. This follows indeed from the mathematical rules of the quantum formalism. But if you give me a million atoms with the same wave function, I can determine the wave function.

Bob: Yes, but I don't give you a million, I give you a single one.

Alice: But it's not clear if the wave function is something *real*. I think it is rather the description of our knowledge about the particle.

Bob: Let's consider this gedanken experiment. Suppose a computer chooses a wave function randomly and prepares an atom with this wave function. Then it prints out some data defining a pair of orthogonal subspaces of the Hilbert space, one of them containing the wave function it had chosen. And then it prints out a note that says *which* of the two subspaces contained the chosen wave function, puts it into an envelope and seals it. After that, the computer erases its knowledge about the wave function. Now, nobody knows the wave function of this atom, and nobody can possibly find out. But Nature still remembers the wave function of this atom, since we can, according to the rules of the quantum mechanical formalism, carry out an experiment that has the two subspaces mentioned earlier as eigenspaces, break the seal and compare the prediction with the actual result. (Strictly speaking, agreement between prediction and result doesn't imply the wave function was contained in one of the subspaces, but the whole procedure can be repeated, and the computer's prediction is *always* true ((unless the quantum formalism is falsified, in which case this dialogue is redundant)). According to the formalism again, the machine can only accomplish certainty of its predictions if the wave function actually lies in the predicted subspace.) So the wave function of the atom is well-defined (or 'known to Nature' or 'real') even in those cases when nobody is aware of it.

Alice: Strictly speaking, you gave an example of one case in which the wave function is well-defined although nobody knows it. This doesn't imply it is always well-defined.

Bob: Strictly speaking, you're right about that. But it suggests that wave functions are always well-defined, and at least it shows that the wave function is not merely a mathematical expression of the observer's knowledge. And it shows there exist things we can't observe.

Alice: If I understood you properly, what you're emphasizing is we can't directly check Schrödinger's equation in the manner of (i) measuring the wave function (without disturbing it), (ii) letting it evolve an amount of time, (iii) measuring the wave function again and (iv) comparing the result with a numerical extrapolation using Schrödinger's equation.

Bob: Yes. Isn't that true?

Alice: It is. And you're saying I shouldn't complain about invisible trajectories as long as I accept Schrödinger's equation.

Bob: You can put it that way. You can, of course, test both eq.s (1) and (2) by their corollaries of second or third generation.

Alice: But how do I know the correct description of reality is Bohmian mechanics rather than any other interpretation of quantum mechanics?

Bob: There is hardly any other interpretation that is consistent, accepts the existence of an outside reality and agrees with the predictions of the quantum formalism (for discussions of other interpretations, see [5] and [8]). To underline this, I'll try to give a sort of 'derivation' of Bohmian mechanics from the formalism. The formalism states that the wave function evolves according to Schrödinger's equation unless we perform a 'formal measurement'. Every 'formal measurement' is characterized by a self-adjoint operator, the possible 'measurement results' are the eigenvalues (in an appropriate, generalized sense) of this operator, the probability of a certain result is the norm square ...

Alice: ... of the projection of the wave function to the corresponding eigenspace, and this projection is the new wave function that remains after the 'formal measurement'.

Bob: Note that there is an ambiguity in the formalism as it is not completely clear which processes are 'formal measurements'. We might either guess the wave function of the measurement apparatus, use Schrödinger's equation for calculating the wave function of the composite system (object + apparatus) after the measurement and *then* invoke the collapse rule when reading off the pointer position (or computer printout or ... ), or we might guess the self-adjoint operator corresponding to this apparatus and right away assume a collapse of the object wave function.

Alice: It is well known and easy to show that this ambiguity does not influence the set of possible results nor their probabilities or probabilities for future 'formal measurements', hence the formalism is unambiguous.

Bob: In so far as macroscopic predictions are concerned. But since we saw that the wave function (of the composite system) is well-defined in reality, the question arises: when does the wave function collapse in reality? It is hard to believe that the universe switches off the 'natural' evolution law for a moment in favor of a different dynamics collapsing the wave function, so apparently the wave function *never* collapses. In this case, however, the wave function of the composite system will, in general, be a superposition of very different states, including different laboratory protocols or whatever (cf. eq. (4)). Especially, the result is not encoded in this wave function. Nor is there any randomness appearing.

Therefore, the wave function cannot be the complete description of the state of the composite system. There have to be additional variables which contain the actual result of the formal measurement. Such variables are usually called 'hidden variables' because they're not part (or functionals) of the wave function; but this name turns out a misnomer if you remember that these variables contain the visible result, the only thing visible, in fact. Now the question is: what are these additional variables? Let's see what the formalism suggests: the wave function is a function of the configuration, i.e. of the particle position. So what's simpler than assuming that 'particles' means particles and that a configuration actually exists? Indeed, what would be the meaning of the wave function being a function of the particle position if there are no particle positions? If we assume that quantum particles have trajectories, too, then the motion of these particles should be guided by the wave function. The precise formula of eq. (1) can be guessed from requiring Galilei invariance of the theory (see [6]).

Alice: I think that whoever says orthodox perception of quantum mechanics is wrong should explain where mistakes were made on the way leading to this perception.

Bob: The founders of quantum mechanics were much attracted by the thought that the words 'momentum', 'energy' and 'angular momentum' still have a meaning in quantum mechanics. These words, however, don't have an immediate meaning (in contrast to 'position' which does); their meaning in Newtonian mechanics comes from the fact that they are conserved quantities. Without this fact, nobody would be interested in multiplying mass and velocity. Now Newtonian mechanics has turned out wrong, so naively we should expect that these words cease to have a meaning. But Heisenberg and others insisted they have a meaning. Heisenberg thought it wise to introduce a physical quantity by giving a definition how to measure it [9]. I don't deem this to be wise because you don't know whether your result depends on the details of your measurement arrangement. There's no problem with defining a quantity by defining how to measure it as long as you can predict the values. Then you can assure the value didn't depend on the arrangement. But there is a problem as soon as the values are random. You don't even know you measured anything meaningful, since whatever definition-how-to-measure you choose, it will always produce *some* result.

And it is interesting which definitions Heisenberg chose: the definitions he gave were always such that in a Newtonian world, they would have measured the Newtonian value (of momentum, energy or angular momentum, respectively). Isn't that strange? Shouldn't we fear that the correct experimental arrangement for measuring momentum (if such a quantity exists) in a world whose rules differ from Newton's might differ from that in Newton's world? Clutching to the belief that Newtonian momentum (energy, angular momentum) measurements reveal *the* momentum (energy, angular momentum) leads to the orthodox view of quantum mechanics.

Alice: Is there an 'actual momentum' in Bohmian mechanics like the 'actual position'?

Bob: You might define $m\dot{Q}$ as the 'actual momentum' (but that is not a conserved quantity), or you might define $\langle\psi|(-i\hbar)\nabla|\psi\rangle$ as the 'actual momentum' (which is a conserved quantity as long as translation invariance is satisfied). But I don't know if such a definition will help for calculations or anything.

Alice: There is a pretty symmetry in quantum mechanics between position and momentum. Bohmian mechanics destroys that symmetry.

Bob: There is no such symmetry in quantum mechanics. The Hamiltonian breaks it. The Schrödinger equation is a differential equation in the position representation of the wave function, but it is only a pseudo-differential equation in momentum representation and just some operator equation in other bases of Hilbert space.

Alice: But you can choose a basis in Hilbert space. That's the symmetry.

Bob: You may as well Fourier-transform Maxwell fields. I can't see any symmetry there.

## Third Day: Special Issues

Alice: What about spin in Bohmian mechanics?

Bob: Replace the Schrödinger equation by the Pauli equation and eq. (1) by

$$\dot{Q}_i = \frac{\hbar}{m_i} \Im \frac{\sum_s \Psi_s^* \nabla_i \Psi_s}{\sum_s \Psi_s^* \Psi_s}$$

where $s$ is the spin index. It is understood that all functions ($\Psi_s$ and its derivatives) are evaluated at the actual configuration.

Alice: So there is no 'actual spin vector'?

Bob: No. The spin is rather a property of the wave function.

Alice: What about identical particles?

Bob: What about identical particles?

Alice: Well, the wave function has to be antisymmetric (respectively symmetric for bosons).

Bob: OK, let the wave function be antisymmetric.

Alice: Nothing special otherwise? The same equation of motion?

Bob: Nothing special otherwise. The same equation of motion.

Alice: The ground state of the hydrogen atom is a real-valued wave function. So the Bohmian electron does not move. It stands still.

Bob: Yes.

Alice: That's counter-intuitive.

Bob: Counter-intuitive doesn't mean much. It may seem counter-intuitive that, according to Maxwell's theory, in a flex the energy is not transported within the wires but within the insulator. For my part, I don't have too much of intuition about the interior of a hydrogen atom. Perhaps you can explain your intuition to me.

Alice: Well, the nucleus executes a Coulomb force on the electron, and this force should in a stationary state be compensated by some centrifugal force.

Bob: So you mean (Coulomb force) + (centrifugal force) = 0? Well, the Coulomb force is $-e^2/r^2$, and the centrifugal force is, well, in general, it is, ehm, $-m\ddot{x}$, right? So your argument implies $m\ddot{x}$ = (Coulomb force). This is precisely Newtonian mechanics, and we can experimentally test Newtonian against Bohmian mechanics. Bohmian mechanics wins.

Alice: But from quantum mechanics one expects that if particle paths are to make sense, they should be Newtonian.

Bob: The existence of particle paths does not imply Newton's equation. It is a popular [9] false conclusion that all particle paths are Newtonian paths. What you refer to in quantum mechanics is the fact that if a small wave packet stays a small wave packet for a time, its (only roughly defined) 'path' is more or less Newtonian. But this is something different from the Bohmian particle path (which is always and precisely defined).

Alice: OK, I'll give a different example. Suppose a particle is jailed between two impermeable walls. Its wave function is a multiple of $e^{ikx} + e^{-ikx}$, where $k$ is chosen appropriate for the wave function to vanish at the walls. Again, the Bohmian particle stands still.

Bob: Yes.

Alice: But quantum mechanics says the momentum is either $\hbar k$ or $-\hbar k$, so the particle can't be at rest.

Bob: The word 'momentum' doesn't have a meaning.

Alice: But we can measure the momentum.

Bob: How do you measure the momentum?

Alice: Take off the walls and let the particle move freely for an amount of time. Then detect its position. If the amount of time was large enough and the distance between the walls small enough, we know quite precisely the stretch the particle made. Now divide by time and multiply by mass.

Bob: The result of this experiment is perfectly predicted by Bohmian mechanics. The trajectory of the Bohmian particle in your experiment looks like this: it is a smooth curve $t \mapsto x(t)$ which is constant $x(t) = x_0$ before the walls are removed and which is asymptotic to the line $x(t) \approx (\hbar k/m)t$ + const. if $x_0 > 0$ and asymptotic to the line $x(t) \approx -(\hbar k/m)t$ + const. if $x_0 < 0$. [This is not precisely true but to the degree of this gedanken experiment.] Each of these two cases occurs with probability 1/2.

Alice: So the particle slowly accelerates until it reaches velocity $\pm\hbar k/m$?

Bob: Yes.

Alice: But I always imagined the particle going back and forth between the walls, having velocity either $\hbar k/m$ or $-\hbar k/m$, each half of the time.

Bob: That's Newtonian mechanics, and Newtonian mechanics is refuted by experiment.

Alice: But Newtonian mechanics for our experiment makes the true prediction that the particle will, with a certain fixed velocity, move either in $x$ or in $-x$ direction after the walls have been removed. So why should we give up Newtonian mechanics in this case?

Bob: Because it can't cope with other experiments, like double-slit.

Alice: I have another question. You said the wave function is something real. So Bohmian mechanics says the wave function is something like a physical field.

Bob: If you wish to put it that way, yes.

Alice: But physical fields are always functions on three-space, not on configuration space. *Probability densities* are functions on configuration space.

Bob: The Maxwell and the gravity field are functions on three-space, but this doesn't mean every physical field is a function on three-space. It is imaginable to have fields on configuration space, why not? Indeed, I can simulate a Bohmian universe on a computer (by the way, it is very unclear how to simulate an 'orthodox' universe on a computer); now what should, say, intelligent life forms inhabiting this universe ...

Alice: That will be quite a slow simulation, I suppose!

Bob: ... think about physical fields?

Alice: Isn't existence of actual particle positions rather a metaphysical question than a physical one?

Bob: An ancient astronomer might have said the positions of the planets in three-spaces cannot be observed so we should restrict our theories to describing the motion of the planets on the two-sphere, against the background of fixed stars. Such a view would certainly have influenced physics, so it would not have been merely of metaphysical interest. That's why I can't see why the existence of trajectories should not be a physical question.

Alice: But as there is no way of testing Bohmian mechanics against orthodox quantum mechanics experimentally, how do I know the trajectories exist?

Bob: 'Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle?' (J.Bell)

Alice: I'm not sure if this is really an answer to my question. How do you know you have the correct trajectories? How do you know it won't turn out necessary to change the equation of motion one day?

Bob: In fact, I don't. But that's not a tragedy. How do you know Schrödinger's equation is correct?

Alice: It certainly isn't. It's non-relativistic.

Both: But that's not a tragedy.

Alice: Why should a physicist deal with philosophical questions?

Bob: Bohmian mechanics is a differential equation. No philosophy. It's the orthodox view which introduces a number of cryptic philosophical phrases for explaining away the problems of quantum mechanics.

Alice: There's one big objection against Bohmian mechanics: the majority of physicists believes in the quantum orthodoxy.

Bob: A philosopher, engineer, mathematician or chemist might comply with the authority of the majority of physicists. But you are a physicist yourself, so you are in the position to decide for yourself. Decide for yourself.

Alice: A final question: How should we scientifically answer metaphysical questions?

Bob: The debate on Bohmian mechanics rather resembles the debate on the question 'In mechanical terms, what does entropy precisely mean and what does the second law of thermodynamics precisely state?' at the turn of the century [7], than a metaphysical debate. I have to explain this comparison. Look, every physical theory we know is more or less ill-defined. Newton's $1/r^2$ force law is ill-defined as soon as two particles collide, the Lorentz force evaluates the Maxwell field at a singular point, and dozens of other problems. Some of these problems we may safely ignore, some not. Some theories are better-defined than others. My message is that usual quantum mechanics is such ill-defined that you should be dissatisfied with it. Now the question is how to make sense out of the formalism of quantum mechanics. The meaning of entropy was discussed in statistical mechanics, and it is the meaning of quantum mechanics we are discussing now. And Bohmian mechanics is the best way to make sense out of quantum mechanics. If you're wondering what does really happen during quantum processes, Bohmian mechanics is the most natural answer.

## Further Readings

At first, I have to mention that this dialogue cited some formulations coined by John Bell and others without marking them. I didn't want to interrupt the flow of reading with footnotes. Of course in general neither the ideas nor the arguments nor the formulations originate from me; I'm only the composer.

At second be informed that I don't claim to be a spokesman of a group of scientists, so whenever something is illogical in my presentation, it's my fault and not theirs.

At third let me say that this is not a typical discussion between a 'Bohmian' and an 'Orthodox' quantum physicist. This dialogue was designed for didactical criteria, and Alice's role is simply to try and get the point about Bohmian mechanics.

[1] J. Bell: *deBroglie-Bohm, delayed-choice double-slit experiments, and density matrix.* 1980. In: *International Journal of Quantum Chemistry:* Quantum Chemistry Symposium 14 (1980), 155-159. Reprinted in: J.S.Bell: *Speakable and unspeakable in quantum mechanics: collected papers in quantum mechanics* (Cambridge University Press 1987), 111-116.

A short paper in which you can watch how Bohmian mechanics works when solving another paradox. What Bell calls 'deBroglie-Bohm theory' is just what is called 'Bohmian mechanics' here.

[2] K. Berndl, D. Dürr, S. Goldstein, G. Peruzzi, N. Zanghi: *On the Global Existence of Bohmian Mechanics.* Comm. Math. Phys. **173**, 647-673, 1995.

[3] K. Berndl, M. Daumer, D. Dürr, S. Goldstein, N. Zanghi: *A Survey of Bohmian mechanics.* Il Nuovo Cimento **110B**, 737-750, 1995.

Learn about the 1995 state of the art and what questions the 'Bohmians' are dealing with today.

[4] D. Bohm: *A suggested interpretation of the quantum theory in terms of 'hidden' variables: Part I.* Physical Review **85**, 166-179, 1952.

D. Bohm: *A suggested interpretation of the quantum theory in terms of 'hidden' variables: Part II.* Physical Review **85**, 180-193, 1952.

The original paper. But be warned: this paper represents the 1952 state-of-affairs. It contains errors on the behaviour of the solutions of the equation of motion, and it contains speculations which haven't lead anywhere.

[5] D. Bohm, B. Hiley: *The Undivided Universe: An Ontological Interpretation of Quantum Theory.* Routledge & Kegan Paul, London 1993.

A whole book on Bohmian mechanics. You will find pictures of Bohmian paths in there and detailed discussions of special topics.

[6] D. Dürr, S. Goldstein, N. Zanghi: *Quantum equilibrium and the origin of absolute uncertainty.* Journal of Statistical Physics **67**, 843-907, 1992.

A detailed analysis of how to justify the Quantum Equilibrium Principle. 64 pages, so you need leisure for this.

[7] P. Ehrenfest, T. Ehrenfest: *Begriffliche Grundlagen der statistischen Auffassung in der Mechanik.* 1911. In: *Enzyklopädie der mathematischen Wissenschaften, Vol. IV-4, Art. 32.*

This reference is here just because statistical mechanics was mentioned in the dialogue, to indicate which area of statistical mechanics Bob is talking of.

[8] S. Goldstein: *Quantum theory without observers.* Physics today **??**, 1998.

Goldstein compares Bohmian mechanics with other attempts of finding out what quantum mechanics means.

[9] W. Heisenberg: *Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik*, Z. Physik **43**, 172-198, 1927.

You might wish to compare Bohmian mechanics with the orthodox view and with the Copenhagen view of quantum mechanics. For the orthodox view, refer to any textbook on quantum mechanics. Since it isn't completely clear whether there is a difference between the orthodox view and the Copenhagen view (i.e. it isn't clear whether today's textbooks present quantum mechanics in a way the Copenhagen pioneers would have accepted), I thought it safe to have a reference to the spokesman of the Copenhagen view. I'm sorry for referring to a German text; sure there is an English translation, but I don't know where.